UNIVERSITI SAINS MALAYSIA

First Semester Examination
2012/2013 Academic Session

January 2013

EME 451/3 – Computational Fluid Dynamics
[Pengkomputeran Dinamik Bendalir]

Duration : 2 hours
Masa : 2 jam

Please check that this paper contains **FIVE (5)** printed pages and **FOUR (4)** questions before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **LIMA (5)** mukasurat bercetak dan **EMPAT (4)** soalan sebelum anda memulakan peperiksaan.]*

INSTRUCTIONS : Answer **ALL** questions. You may answer all questions in **English** OR **Bahasa Malaysia** OR a combination of both.

*[**ARAHAN :** Jawab **SEMUA** soalan. Calon boleh menjawab semua soalan dalam **Bahasa Malaysia** ATAU **Bahasa Inggeris** ATAU kombinasi kedua-duanya.]*

Answer to each question must begin from a new page.
[Jawapan untuk setiap soalan mestilah dimulakan pada mukasurat yang baru.]

In the event of any discrepancies, the English version shall be used.
[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Q1. Summarize the major mathematical and physical differences between compressible and incompressible flows. Include at least 5 factual points. How are these differences reflected in typical CFD codes/software for these two situations? Include at least 3 factual points.

Rumuskan perbezaan utama matematik dan fizikal antara aliran mampat dan tidak mampat. Sertakan sekurang-kurangnya 5 fakta. Bagaimana perbezaan ini membentuk kod-kod komersial CFD? Sertakan sekurang-kurangnya 3 fakta.

(100 marks/*markah***)**

Q2. The 1D Navier-Stokes equations can be modeled by

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial (\mathbf{f})}{\partial x} = \frac{\partial (\mathbf{f}^v)}{\partial x} \qquad (1)$$

where $\mathbf{u}$ is vector representing the mass, momentum and energy while $\mathbf{f}$, $\mathbf{f}^v$ represents the inviscid and viscous fluxes .

[a] The inviscid part can be discretized using the MacCormack (two-step) scheme

$$\begin{aligned} \mathbf{u}_j^* &= \mathbf{u}_j^n - \frac{\Delta t}{\Delta x}[\mathbf{f}_j^n - \mathbf{f}_{j-1}^n] \\ \mathbf{u}_j^{n+1} &= \frac{1}{2}[\mathbf{u}_j^n + \mathbf{u}_j^*] - \frac{\Delta t}{2\Delta x}[\mathbf{f}_{j+1}^* - \mathbf{f}_j^*] \end{aligned} \qquad (2)$$

where $\mathbf{f}^* = \mathbf{f}(\mathbf{u}^*)$, and $\mathbf{u}^*$ is an intermediate quantity.

Assume that the inviscid part can be modeled by the linear (scalar) advection equation where $\mathbf{u} = u$, and $\mathbf{f} = au$. Apply the linear advection equation in MacCormack scheme in Eqn. (2) and determine what does it reduce to? Determine if this scheme can be used to take into account information coming in both left and right directions when solving the linear advection. Give a reason.

Andaikan bahagian 'inviscid' dimodelkan oleh persamaan (skalar) adveksi linear di mana sas. Masukkan nilai-nilai persamaan adveksi linear dalam teknik MacCormack dalam persamaan (2) dan tentukan apa hasilnya? Tentukan jika teknik yang digunakan ini mengambil kira informasi yang dating daripada arah kiri dan kanan semasa menyelesaikan masalah adveksi linear. Nyatakan sebabnya.

(50 marks/*markah***)**

[b] The viscous part of the Navier-Stokes equations can be modeled using a similar approach as in the inviscid part. Write down the governing model equation for the Navier Stokes which includes both inviscid and viscous parts.

Bahagian likat persamaan Navier-Stokes boleh dimodelkan dengan cara yang sama seperti bahagian 'inviscid'. Tuliskan model persamaan Navier-Stokes yang mengambil kira bahagian 'inviscid' dan bahagian likat.

(15 marks/*markah*)

[c] Determine a suitable discretization method for the model viscous part.

Tentukan persamaan diskrit yang sesuai untuk model bahagian likat.

(15 marks/*markah*)

[d] Determine the stability limit for the discrete model Navier-Stokes equations. Evaluate the dominant limiting factor in stability, for both inviscid part and the viscous part. Justify your answer.

Tentukan had stabil untuk model persamaan diskrit Navier-Stokes. Berikan penilaian faktor kestabilan untuk bahagian 'inviscid' dan likat. Berikan justifikasi anda .

(20 marks/*markah*)

Q3. [a] Grid quality is one of the vital properties that can affect the discretization errors, slow convergence and also numerical instability. Discuss some rules of good computational practices in grid quality justification.

Kualiti grid adalah salah satu perkara penting yang boleh mengubah kesilapan pendiskretan, penumpuan perlahan dan juga ketidakstabilan berangka. Bincangkan beberapa peraturan amalan yang baik dalam justifikasi kualiti grid.

(40 marks/*markah*)

[b] Discuss the disadvantages of the unstructured grids approach which are primarily related to the complexity of the grid description.

Bincangkan kelebihan pendekatan grid tidak berstruktur, terutamanya yang berkaitan dengan kerumitan perihal grid.

(20 marks/*markah*)

[c] **To illustrate the discussion of grid quality, consider the Figure Q3[c]. Both grids are generated using GAMBIT 2.3. The grid (i) is two-dimensional and grid (ii) is three dimensional grid, both are built of quadrilateral and tetrahedral cells, respectively. The grids do not contain topologically bad cells when the grid checking algorithm and grid independent study have been performed. The near orthogonality is evident for grid (i) that consists of nearly rectangular cells. Grid (ii) can be assumed ideal since the tetrahedral cells are nearly equilateral and approximately of the same. Criticize both grids illustrated in Figure Q1[c] below.**

Untuk menggambarkan perbincangan berkualiti grid, pertimbangkan Rajah S3[c]. Kedua-dua grid dijana menggunakan Gambit 2.3. Gambar rajah atas adalah dua dimensi dan yang di bawah pula adalah grid tiga dimensi yang terdiri daripada sel sisiempat dan tetrahedral, masing-masing. Grid-grid tersebut tidak mengandungi topologi sel buruk apabila pemeriksaan grid algoritma dan kajian grid bebas telah dijalankan. Keortogonan berhampiran adalah jelas untuk grid atas yang terdiri daripada sel-sel yang hampir segiempat. Grid yang lebih rendah boleh dianggap ideal sejak sel tetrahedral adalah hampir sama sisi dan kira-kira yang sama. Buat kritikan ke atas kedua-dua grid yang ditunjukkan dalam Rajah S3[c] di bawah.

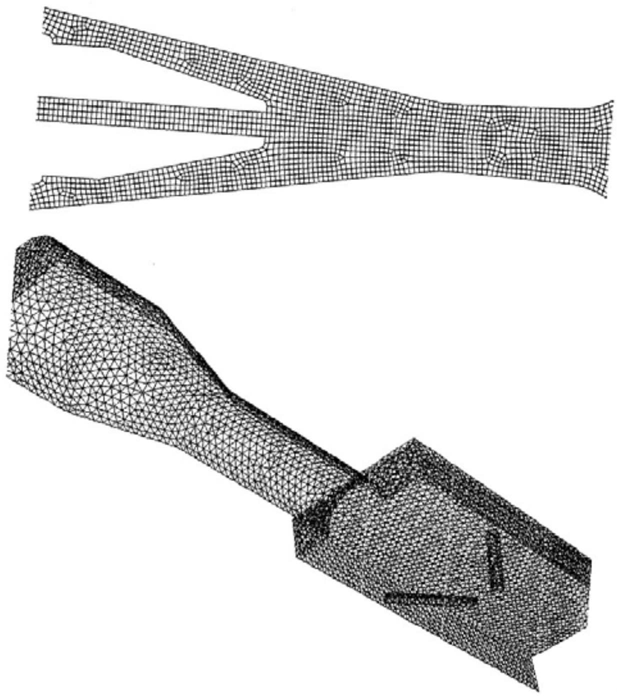

Figure Q3[c]
Rajah S3[c]

(40 marks/*markah*)

Q4. [a] **Consider a primitive LES or RANS model, in which the eddy viscosity formula given as the following;**

Pertimbangkan LES primitif atau RANS model, di mana formula kelikatan pusar diberikan sebagai berikut;

$$\boldsymbol{\tau_{ij} \equiv \rho\left(u_i' u_j'\right) = -2\mu_t\left(S_{ij}\right) + \frac{2}{3}\rho\delta_{ij}k}$$

$$\boldsymbol{\tau_{ij}^r = -2\mu_t\overline{S}_{ij}}$$

are applied with constant eddy viscosity μ_t. Discuss the drawback of this model explaining whether we should expect accurate results.

digunakan dengan μ_t kelikatan eddy berterusan. Bincangkan kelemahan model ini dengan menerangkan sama ada keputusan tepat akan diperolehi.

(40 marks/*markah***)**

[b] If turbulence is inhomogenous in one or several directions, for example if the flow domain has realistic boundaries, such as solid walls, the spectral method based on the three dimensional Fourier expansion cannot be used. Discuss what approach can be of alternatives?

Jika pergolakan inhomogenous dalam satu atau beberapa arahan, sebagai contoh jika domain aliran mempunyai sempadan yang realistik, seperti dinding pepejal, kaedah spektrum yang berdasarkan tiga dimensi Fourier pengembangan tidak boleh digunakan. Bincangkan apakah pendekatan yang boleh menjadi alternatif?

(30 marks/*markah***)**

[c] Take a classical example of fully developed turbulent flow in a channel and assume the turbulent is homogenous. Explain how the Fourier expansion equation can be solved. Include also in your explanation the grid near the wall treatment on how the Navier-Stokes equations come into the transformation in x- and y-coordinates.

Ambil satu contoh klasik aliran maju bergelora dalam saluran dan menganggap aliran gelora adalah homogenus. Terangkan bagaimana persamaan Fourier pengembangan boleh diselesaikan. Jelaskan grid berhampiran rawatan dinding serta bagaimana penjelmaan persamaan Navier-Stokes dalam x dan y-koordinat boleh dimasukkan.

(30 marks/*markah***)**

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